9. Storm Surges

9.1 Introduction

In September 1999, a large typhoon generated storm surges in Yatsushiro Bay, Kyushu. On the coast of Mitsuhashi town, sea water reached the 5-meter (above mean sea) level, and there were six casualties. The typhoon also generated storm surges in the western part of the Seto Inland Sea, and the runway of the Onoda Airport was covered by sea water up to a depth of 9 m; parked cars were swept into the ocean.

In 1934, Typhoon "Muroto," whose central atmospheric pressure was less than 900 Hpa, induced huge storm surges in Osaka Bay. The central part of Osaka city was inundated up to a height of 3.5 m.

In 1964, the Ise-wan Typhoon hit Nagoya city and its vicinity, and more than 5,000 people were killed by the accompanying storm surges.

Today, the coasts of Japan are fully equipped to deal with such hazards, and small storm surges have become events of the past.

However, this belief has sometimes been proved wrong. For example, in 1992, Typhoon 9219 passed Kyushu-Island and the western part of Seto Inland Sea; the residential area of Hiroshima was submerged up to the ground level of 2.2 m and 23 people died. Evenin the modern era, storm surges are real hazards in Japan.

9.2 Abiki Phenomenon

At several ports on the western coast of Kyushu Island, for example, Nagasaki Bay and Makurazaki Bay, it is well known that extra-ordinarily large standing waves or proper oscillations (Seiche) are sometimes induced; they are generally referred to as the Abiki phenomena, and it causes considerable damage to fishery boats. Such types of proper oscillations are not always induced on a stormy day; in fact, the Abiki phenomenon occurrs on during days of mild weather. Hibiya and Kajiura (1982) clarified that this phenomenon is induced by a kind of resonance between the velocity of a long wave and the speed of an atmospheric shock wave. This phenomena has been reported only at harbors on the western coast of Kyushu.

9.3 Factors Influencing the generation of Storm Surges

Storm surges are a phenomenon where sea water floods a plain on a stormy day. What are the physical factors causing storm surges?

1. Atmospheric pressure drop
2. Increase in the amount of sea water due to wind

3. Increase in sea water amount induced by the mass transport by non-linear wave effects (Stokes waves, Cnoidal waves, or Soliton)
4. Resonance between wave and cyclone speeds

Factors 1, 2, and 3 can be understood by relatively uncomplicated physics, but Factor 4 is relatively complicated.

1. Atmospheric pressure drop

The standard atmospheric pressure at 0 m asl (sea level) is generally 1013 Hpa. Hence, storm surges occur in a cyclonic area where the atmospheric pressure is below 1013 Hpa.

Sea water is sucked due to the decrease in the atmospheric pressure, and the amount of increase in the sea surface ΔH_P is given by

$$\Delta H_P = (P - 1013.0) \times 0.991 \qquad (9.1)$$

where P is the atmospheric pressure in Hpa. For example, for a typhoon whose central pressure is 930 Hpa (a strong typhoon), 81 cm of sea water will be sucked. Such an increase in the sea level cannot cause severe damage to the coast; slight damage to fishing boats may occur.

2. Increase in the amount of sea water due to wind

The effect of the amount of increase in sea water due to wind was discussed by Colding who developed a statistical discussion on the basis of the accumulated data from the North Sea. He pointed out that sea water would increase due to the tangential stress τ by wind, and that the sea surface inclination I is proportional to the square of the wind speed and inversely proportional to the depth. He obtained the following empirical formula:

$$I = k\frac{(U\cos\theta)^2}{D} \qquad (9.2)$$

where U is the wind speed (unit: m/s, 10-min average); θ, the angle between the bay axis and wind direction; *D,* the water depth (m), and I, the inclination of the sea surface induced by the wind. k is a constant and Colding gave its value as 4.0×10^{-7}. We assume that the length of a bay axis is given by λ (km) and the increase in the sea surface at the bay mouth is zero; the height of the sea surface increasing at the innermost point of the bay ΔH_{WC} is then given by

$$\Delta H_{WC} = k\lambda\frac{(U\cos\theta)^2}{D} \qquad (9.3)$$

Tokyo bay has an axis length of 70 km, and the average depth is about 20 meters.

On October 19, 1979, Typhoon #20 hit Kanto District. Strong winds of more than 30 m/s (10-min average) were observed at Tokyo and Yokohama. In Table 9.1, the observed sea surface rise and the pressure and wind factor are shown.

Table 9-1 Sea surface rise (abnormality) at the tide gauge stations on the coast of Tokyo Bay caused by typhoon 7920.

Stations	Tidal abnormality	Pressure Factor	Wind Factor	Colding's formula
	ΔH	ΔH_P	ΔH_W	ΔH_{WC}
Aburasubo	37 cm	33 cm	4 cm	—
Yokosuka	57 cm	34 cm	23 cm	20 cm
Yokohama	79 cm	34 cm	45 cm	32 cm
Tokyo-Harumi	114 cm	33 cm	81 cm	83 cm
Chiba	104 cm	32 cm	72 cm	106 cm

Since the inclination is inversely proportional to the depth according to Colding's formula (19), we can expect the sea surface rise ΔH_{WC} at the bay mouth would be almost nil. This is also because the depth of an open ocean is generally more than that of an inner bay area.

Note that although Colding's formula is effective as an empirical formula by which we can estimate the effect of the increase in the sea water, this formula does not deal with the physical process of storm surges.

9.3 Factors Amplifying Storm Surges

We have discussed that decrease in the atmospheric pressure and increase in sea water due to drag force by wind are basic factors Influencing sea surface rises. However, on the other hand, note that we still cannot explain why sea water increased up to 3 to 5 m in the case of the Muroto (1934) and Isewan Typhoons only as a result of these basic factors. To explain such extra-ordinarily large storm surges, we should also consider the process of amplification.

We can consider the following processes:

(1) Velocity resonance that sometimes occurs when the velocity of the typhoon is close to the long wave speed.

(2) Resonance of inner and outer bays that occurs when the periods of the proper oscillations of the inner and outer bays are close

(Nagasaki bay and the Goto-Nada sea)

(3) Shelf wave resonance: when the periods of the fundamental mode of an inner bay and that of a mode of continental waves are close (Makurazaki Bay)

(4) Resonance of passing time of a typhoon in a bay and natural oscillation period of the bay: When the periods of the fundamental mode of the proper oscillation and time scale = (typhoon radius) / (typhoon speed) are close to each other.

(5) Amplification at the innermost point of a V-shaped bay

(6) Amplification around an isolated island with a large skirt sea area

(1) ~ (4) refer to amplifications caused by resonance.

The most reasonable and important amplification factor is (1), which we will discuss in the next section.

9.4 Generation of Storm Surge due to Resonance between typhoon and long wave speeds

In this section, we discuss the displacement of a sea surface when a moving cyclone passes a coast.

We assume that the depth of the sea, D, is a constant and that a typhoon moves at a constant speed V. We take the x-axis along the direction of the typhoon's route, and the y-axis in perpendicular to it. We assume that for a person moving at the same speed as the typhoon, the sea surface is still. We introduce a velocity potential ϕ thar satisfies the following relation:

$$(u,v,w)=\left(\frac{\partial\phi}{\partial x},\frac{\partial\phi}{\partial y},\frac{\partial\phi}{\partial z}\right)$$

The condition of the mass conservation takes the following form:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad (9\text{-}4)$$

The dynamic (pressure) surface condition within the linear approximation is given by

$$-\frac{\partial\phi}{\partial t}+g\zeta=-\frac{p}{\rho} \qquad (9\text{-}5)$$

Here ζ is the displacement of the sea surface and p is the atmospheric pressure.

Note that the pressure is not a constant in the present problem.

The condition for a sea bed is

$$w=0 \quad \text{on} \quad z=-D \qquad (9\text{-}6)$$

The form of the velocity potential satisfying (11-4) and (11-6) and the stationary condition $\phi=f(x-Vt)$ has the following form

$$\phi = Ce^{i(\alpha x+by)-i\alpha Vt} \cosh k(z+D) \qquad (9\text{-}7)$$

Here $\alpha^2 + \beta^2 = k^2$ should be satisfied.

(Note : x is contained only in the function of $(x-Vt)$, which means that φ is a constant for an observer moving in a car with a speed V in the positive x direction.)

If we differentiate $\cosh k(z+D)$ twice wrt z, we have $k^2 \cosh k(z+D)$. On the other hand, differentiating $e^{i(\alpha x+\beta y)}$ twice wrt x, y yields the coefficients $-\alpha^2, -\beta^2$ t. Therefore if $\alpha^2 + \beta^2 = k^2$ is satisfied, condition (11-4) is satisfied.

From the condition for sea bed, should obtain $\partial\phi/\partial z = 0$ for $z = -D$. If we differentiate $\cosh k(z+D)$ wrt z, we obtain $k \sinh k(z+D)$; here, we can recognize that if we set $z = -D$), it reduces to zero.

The kinematic water surface condition in the linear approximation is

$$\frac{\partial \zeta}{\partial t} = w(= -\frac{\partial \phi}{\partial z}) \qquad (9\text{-}8)$$

Substituting (11-7) in (11-8), we have

$$\zeta = \left[\int \left(-\frac{\partial \phi}{\partial t} \right) dt \right]_{z=0} = -\frac{ik}{\alpha V} Ce^{i(\alpha x+\beta y)-i\alpha Vt} \sinh kD \qquad (9\text{-}9)$$

(11.9) is a single-element solution for one set of (α, β); however, in reality, we can assume that the entire solution is expressed by the addition of a full set for the cases by changing α, β. We regard the constant C in (9.9) as a function of α, β, and we write $C(\alpha, \beta)$. The total solution can then be expressed as a linear combination of α,β in the form of a double integration of (α, β) as follows:

$$\phi = \iint_{\alpha,\beta} C(\alpha, \beta) e^{i(\alpha x+\beta y)-i\alpha Vt} \cosh k(z+D) d\alpha d\beta \qquad (9\text{-}9b)$$

$$\zeta = -i \iint_{\alpha,\beta} C(\alpha, \beta) \frac{k}{\alpha V} e^{i(\alpha x+\beta y)-i\alpha Vt} \sinh kD d\alpha d\beta \qquad (9\text{-}9c)$$

We assume the distribution of the atmospheric pressure in a cyclone to be $p_0(x-Vt, y)$. From the theorem for two--dimensional double Fourier transformation, the following equation can be used for any function $p_0(x, y)$

$$p_0(x-Vt, y) = \frac{1}{4\pi^2} \iint_{\alpha,\beta} e^{i(\alpha x+\beta y)-i\alpha Vt} d\alpha d\beta \iint_{\xi,\eta} p_0(\xi, \eta) e^{-(\alpha\xi+\beta\eta)} d\xi d\eta \qquad (9\text{-}10)$$

On the other hand, the pressure in water just below the surface is given by (9-5)

$$p_0 = \rho\left\{\left[\frac{\partial\phi}{\partial t}\right]_{z=0} - g\zeta\right\} \tag{9-11}$$

Substituting (9-9b) and (9-9c) into this will give

$$p_0(x-Vt,y) = -i\rho\iint_{\alpha,\beta} C(\alpha,\beta)\left(\alpha V\cosh kD - \frac{gk}{\alpha V}\sinh kD\right)e^{i(\alpha x+\beta y)-i\alpha Vt}\,d\alpha d\beta \tag{9-12}$$

By making (9-10) equal to (9-9), we obtain $C(\alpha,\beta)$ in the following form:

$$C(\alpha,\beta) = i\frac{1}{4\pi^2\rho\alpha V\cosh kD\left\{1-\dfrac{gk}{\alpha^2V^2}\tanh kD\right\}}\iint_{\xi,\eta} p_0(\xi,\eta)e^{i(\alpha\xi+\beta\eta)}\,d\xi d\eta \tag{9-13}$$

By substituting $C(\alpha,\beta)$ in (11-9b), we obtain the water surface displacement function ζ as follows:

$$\zeta = \frac{1}{4\pi^2\rho}\iint_{\alpha,\beta}\frac{k\sinh kD}{\alpha^2V^2\cosh kD\left(1-\dfrac{gk}{\alpha^2V^2}\tanh kD\right)}e^{i(\alpha x+\beta y)-i\alpha Vt}\,d\alpha d\beta\iint_{\xi,\eta} p_0(\xi,\eta)e^{-i(\alpha\xi+\beta\eta)}\,d\xi d\eta \tag{9-14}$$

Given the atmospheric pressure distribution of a typhoon $p_0(x,y)$, we can calculate the sea surface displacement $\zeta(x,y)$ by using (9.14).

Hereafter, we assume the pressure distribution in a typhoon in its most simple form as follows:,

$$p_0(x,y) = \begin{bmatrix} 0\ldots\ldots x^2+y^2 > a^2 \\ P\ldots\ldots x^2+y^2 \le a^2 \end{bmatrix} \qquad (P: \text{ a constant}) \tag{9-15}$$

Then, the double integration I for ξ,η in (11-15) becomes simply

$$I = \iint_{\xi,\eta} p_0(\xi,\eta)e^{-i(\alpha\xi+\beta\eta)}\,d\xi d\eta = P\iint_{\xi^2+\eta^2\le a^2} e^{-i(\alpha\xi+\beta\eta)}\,d\xi d\eta \tag{9-16}$$

We transfer the subordinate variables (α,β) into $\alpha = \frac{s}{a}\cos\vartheta, \beta = \frac{s}{a}\sin\vartheta$, and subordinate variables (ξ,η) into $\xi = \sigma\cos(\vartheta+\varphi), \eta = \sigma\sin(\vartheta+\varphi)$. Jacobian function for the transform $(\xi,\eta)\to(\sigma,\varphi)$ is given by

$$J\frac{(\xi,\eta)}{(\sigma,\varphi)} = \begin{vmatrix} \dfrac{\partial\xi}{\partial\sigma} & \dfrac{\partial\eta}{\partial\sigma} \\ \dfrac{\partial\xi}{\partial\varphi} & \dfrac{\partial\eta}{\partial\varphi} \end{vmatrix} = \begin{vmatrix} \cos(\vartheta+\varphi) & \sin(\vartheta+\varphi) \\ -\sigma\sin(\vartheta+\varphi) & \sigma\cos(\vartheta+\varphi) \end{vmatrix} = \sigma$$ Hence, the integral in

(9-16) becomes

$$I = P \iint_{\xi^2+\eta^2 \le a^2} e^{-i(\alpha\xi+\beta\eta)} d\xi d\eta = P\int_0^a \left\{ \int_{-\pi}^{\pi} e^{-i\frac{s}{a}\sigma\cos\varphi} d\varphi \right\} \sigma d\sigma$$

$$= 2\pi P \int_0^a J_0\left(\frac{s}{a}\sigma\right)\sigma d\sigma = 2\pi a^2 P \frac{J_1(s)}{s} \qquad (9\text{-}17)$$

where $J_1(s)$ is Bessel function of the first order.

For the integration in (9-16), we transfer (x, y) to the polar coordinate system (r,θ) by using the relationships $x = r\cos\theta, y = r\sin\theta$. Further, we use $r/a = \omega$ ($\omega < 1$ implies an area inside the cyclone.)

$$\zeta = \frac{P}{2\sigma\rho g}\int_0^{2\pi}\int_0^{\infty} \frac{J_1(s)}{\left(\frac{r^2\cos^2\phi}{gD} - 1\right)} e^{is\omega\cos(\theta-\phi)} ds d\phi \qquad (9\text{-}18)$$

We introduce the Bessel series expansion for the exponential part in (9-18).

$$e^{is\omega\cos(\theta-\phi)} = J_0(\omega s) + 2\sum_{n=1}^{\infty} i^n J_n(\omega s)\cos^n(\theta-\phi) \qquad (9\text{-}19)$$

Then, (9-18) becomes

$$\zeta = \frac{P}{2\pi\rho g}\left[\int_0^{\infty} J_1(s)J_0(\omega s)ds \int_0^{2\pi} \frac{1}{\frac{V^2}{gD}\cos^2\phi - 1} d\phi + \sum_{n=1}^{\infty} i^n \int_0^{\infty} J_1(s)J_n(\omega s)\int_0^{2\pi} \frac{\cos n(\theta-\phi)}{\frac{V^2}{gD}\cos^2\phi - 1} d\phi\right] \qquad (9\text{-}20)$$

We put the integral parts for ϕ in (9-20) as

$$I_n = \int_0^{2\pi} \frac{\cos n(\theta-\phi)}{q^2\cos^2\phi - 1} d\phi \qquad (q^2 = V^2/gD) \qquad (9\text{-}21)$$

When $q^2 < 1$, that is, when the moving velocity of the cyclone V is less than the long wave velocity ($V < \sqrt{gD}$), we can re-write the integral (9-21) as

$$I_n = \frac{1}{2}\left(\int_0^{2\pi} \frac{\cos n(\theta-\phi)}{q\cos\phi - 1} d\phi - \int_0^{2\pi} \frac{\cos n(\theta-\phi)}{q\cos\phi + 1} d\phi\right) \qquad (9\text{-}22)$$

When n is an odd number, this integral becomes zero, while when it is even, it is given as follows:

$$I_n = -\frac{2\pi}{\sqrt{1-q^2}} z_1^n \cos n\theta \tag{9-23}$$

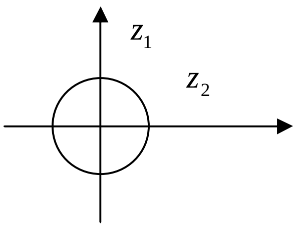
Here z_1 is one of the solutions of the following quadratic equation

$$z^2 - \frac{2}{q} z + 1 = 0 \tag{9-24}$$

which satisfies $|z| < 1$.

[Problem] Show that I_n is given as above mentioned.
Hint: We set $z = e^{i\phi}$ and then

$$\int_0^{2\pi} \frac{\cos n(\theta-\phi)}{q\cos\phi - 1} d\phi = -\frac{2i}{q} e^{-in\theta} \oint \frac{z^n}{z^2 - \frac{2}{q} z + 1} dz \tag{9-25}$$



z_1, z_2 are the solutions of

$$z^2 - \frac{2}{q} z + 1 = 0$$

We finally obtain (9-20) as

$$\zeta = \frac{P}{\rho g} \frac{1}{\sqrt{1-q^2}} \int_0^\infty J_1(s) J_0(\alpha s) ds - \frac{2P}{\rho g} \frac{1}{\sqrt{1-\xi^2}} \sum_{m=1}^{\infty} (-1)^m \cos 2m\theta z_1^{2m} \int_0^\infty J_{2m}(\alpha s) J_1(s) ds$$

(9-26)

I. For inside the cyclone $(\alpha < 1)$

$$\int_0^\infty J_0(\alpha s) J_1(s) ds = 1, \int_0^\infty J_1(s) J_{2m}(\alpha s) ds = 0 \tag{9-27}$$

Finally, we have

$$\zeta = -\frac{P}{\rho g} \frac{1}{\sqrt{1-q^2}} = -\frac{P}{\rho g} \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \tag{9-28}$$

where $c^2 = gD$.

II. For outside the cyclone, $\omega > 1$ and (9-27) does not hold. However, ζ takes the following form:

$$\zeta = \frac{2P}{\rho g}\frac{1}{\sqrt{1-q^2}}\frac{z_1^2}{\omega^2}\left\{\cos 2\theta - 2\left(1-\frac{3}{2\omega^2}\right)z_1^2\cos 4\theta + 3\left(1-\frac{4}{\omega^2}+\frac{10}{3\omega^4}\right)\cos 6\theta + \ldots\right\}$$

(9-29)

The numerically calculated result of (9-29) shows that small hollows appear in the front and rear areas of a cyclone while small hills (wings) spread in both right and left sides of the cyclone.

The absolute values of the hollows and wings are so small When compared with the sucking inside the cyclone that we can neglect them actually.

Thus we can conclude that

(1) The amount of water rising (sucked) is uniform in the cyclone area

(2) Further, this amount is $1/\sqrt{1-V^2/c^2}$ times that of the increase in the static pressure $\varsigma_0 = -P/(\rho g)$

(Notice: According to Einstein's Tof Relativity, the mass of moving matter increases in the form of $m/\sqrt{1-V^2/c^2}$; this resembles (9-28))

When the speed of a cyclone V is close to the long wave speed $c(=\sqrt{gD})$, the denominator in (9-28) becomes large and infinite, which implies a type of resonance between the cyclone and long wave velocities.

The average speed of a cyclone is generally around 50 km/h, while the long wave speed in Yellow Sea (average depth of approximately 100 m) between the Chinese Mainland and Kyushu is

$$c = \sqrt{gD} = \sqrt{9.8\times 100} = 31.0 \text{ m/s} = 19 \text{ km/h}.$$

Hence, even in this shallow sea, this type of resonance occurs rarely.

The depth in Tokyo and Osaka Bays is approximately 20m; hence, $c =$ 14.0 m／s ＝ 50.4 km／h. Resonance is possible here, but the sea area is too small.

Hibiya and Kajiura (1982) pointed out that a type of atmospheric front waves move across the Yellow Sea at speeds of 110 km/h. Such waves can cause such resonance, and they are a probable cause of the “Abiki Phenomena” occurring in Nagasaki Bay.